Cognitively Guided Instruction:

The Inquiry Learning of Mathematics!

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Abstract

This paper describes an instructional approach to mathematics in the primary grades. The approach, based on twenty years of research, is called Cognitively Guided Instruction. It is an approach grounded in the philosophy of constructivism, the cognitive developmental stages of Piaget, and the process standards set forth by the National Council of Teachers of Mathematics. Other benefits of CGI are the ease with which there is continual assessment of children's cognitive strategies, and the ease with which the teacher can scaffold the children's learning. A common theme found throughout this instructional approach is the focus on being an active participant in constructing one's learning. It is the same theme found in Inquiry Learning, a constructivist approach that applies the Scientific Method to real life.

Cognitively Guided Instruction: The Inquiry Learning of Mathematics!

Cognitively Guided Instruction, developed under the leadership of Tom Carpenter and Elizabeth Fennema at the University of Wisconsin the 1980's and 1990's, is an instructional approach for learning mathematics in, primarily, grades K – 3. The approach is based on the hypothesis that it is important for teachers "to know the mental processes, or cognitions, by which learners acquire specific subject matter knowledge in mathematics" (Peterson, Fennema, & Carpenter, 1989). In other words, continual assessment is an integral part of this approach to mathematics instruction. Teachers are able to assess what mental strategies a child uses to solve a problem by asking the child questions about how he/she came up with the answer. The language used by the teacher is crucial – both in how the problem is stated, and in how the assessment questions are voiced. The wording of the problem is important for two reasons. First, different wordings provide different levels of difficulty, and thus provide clues to the child's level of cognitive understanding and skill. As a result, the wording of a problem allows the teacher to scaffold the child's level of understanding and skill to higher levels. Scaffolding allows the bridge from what the child already knows to what is just beyond his/her reach. Inherent in the philosophy of CGI is that "learning is the making of connections between new information and the learner's existing network of knowledge – the construction of knowledge by the learner" (Peterson, Fennema, & Carpenter, 1989).

Twenty years of research went into the development of Cognitively Guided Instruction, and the research showed that children enter kindergarten with a great deal more problem solving strategies than adults expected. "Young children are naturally curious and have a desire to make sense of their world. In their early experiences they encounter a variety of situations involving quantities, and at a very early age they begin to recognize relationships involving those quantities. By the time they begin school, most children have started to learn to count and demonstrate remarkable insight about how to use their emerging counting skills to solve problems" (Carpenter, Fennema, Franke, Levi, & Empson, 1999). To young children, figuring out a way to solve a problem is simply part of life. Counting is part of life. If a child has one train car, and the parent gives him/her two more train cars, the child may count his new number of train cars and realize that he used to only have one car, but now he has three! If there are only two cookies, and the child has a sister who also wants a cookie, the child will begin to realize that if he/she ate both cookies, his/her sister wouldn't get one! Or, if there are only two cookies, and there are three kids, the kids may figure out on their own that there aren't enough cookies for everyone to have one! Simply faced with the experiences of real life, the child will learn strategies for problems that involve addition, subtraction, multiplication, and division. If, when a child enters school, the experience and personal meaning are taken out of the problem, and math becomes simply facts for rote memorization, such as 3 + 3 and 7 – 2, the child will lose his/her understanding of how numbers make sense.

By giving children the freedom to solve a story problem in any way they choose, CGI allows a child to use the knowledge that he/she already has. As different children come up with different strategies to solve the problem, and share with each other how they did it, children are able to hear and see the strategies used by others. Through this process of sharing, children make connections between how they chose to solve the problem and how others chose to solve it. These connections result in learning. Sharing, then, is an important part of Cognitively Guided Instruction.

The progression of strategies used by children for solving problems coincide with the developmental stages of Piaget. The strategies progress from concrete to abstract. A concrete strategy is directly modeling the action in the problem. For example, if the problem states that Sue has two tennis balls and finds three more tennis balls, the child may count out two blocks, and than count out three more blocks, and then push them together and count all five blocks. The child is *joining all* or *adding all* the addends. As the child's cognitive skills develop, he/she may move from the blocks to using his/her fingers to count. If he/she holds up two fingers on one hand and three fingers on the other, and counts all five fingers, he/she is still *joining all*. When he/she begins to count from the two, he/she is now using a more advanced strategy. He/she is *counting on from the first addend*. Eventually, he/she will see that it is also possible to *count on from the larger number*. For example, if the problem states that Sayumi has three worms for fishing, and her brother gives her 22 more worms, one child may count on from three (three…four, five, six, seven, eight, …twenty-five.) In time, the child will learn that he/she could also just count on from the 22 (twenty-two…twenty-three, twenty-four, twenty-five.) As the child continues to experience more of these problems, and as his/her cognitive skills continue to develop, he/she will begin to remember number facts and use them in solving the problems. Some number facts are more quickly and easily learned than others. The doubling of numbers, and the numbers which total ten, are the some of the first facts a child will master. The knowledge of these number facts will add flexibility and inventiveness to the strategies the child uses. For example, if a problem states that there are seven chickens in the barnyard, and nine more chickens join them, the child may immediately say there are 16 chickens. If asked how the child came up with that number so quickly, he/she may say that 7 and 7 is 14, and 2 more is 15. Or, the child may say that 7 and 3 is 10, and 6 more is 16. Or, the child could say that he/she took one from the 9 and gave it to the 7, and 8 and 8 is 16. These children are solving the problem using *derived facts* – those that they already know. (Carpenter, Fennema, Franke, Levi, & Empson, 1999).

The strategy the child chooses depends not only on his/her level of cognitive development, but also on how the problem is worded. For example, one problem may state that there are two birds sitting on a tree. Three more *join* them. How many are there? A second problem may state that there are two birds sitting on a tree. Some more birds *join* them and then there are five. How many birds joined them? A third problem may state that some birds are sitting on a tree. After 3 more birds *join* them, there are five birds on the tree. How many birds had been there in the beginning? Each of these three problems involves the same computation: 2+3=5. The first addend is how many birds there were to start with. It is the starting number. The addition of a new addend changes the total number. The second addend can be referred to as the *change*. When the amounts of the two addends are joined together, there is a new *result*ing number. In the first problem, the *result* is unknown: 2+3= . In the second problem, the *change* is unknown: 2+ =5. In the third problem, the starting number is unknown: +3=5. The wording of the problem changes the difficulty level of the problem, and it changes the strategy the child will use. If the child doesn't know both of the addends, he/she can't add them all up. He/she will have to come up with another strategy. The most difficult wording is the problem in which the starting number is unknown. The reason this is most difficult is because children who solve problems by concrete means are following the action of the story problem. When a problem says there are two birds in a tree, the child counts two blocks. When the problem says that three more birds join them, the child counts out three more blocks. In the case of a problem with the start unknown, where there are just *some* unknown number of birds sitting on a tree, the child doesn't know how many blocks to count out. If this problem is too hard for a child to solve, the teacher can assess the child's cognitive level of understanding and skill to be not quite there yet, and change the wording to a level that the child is able to succeed at. The wording, the teacher language, is the key to scaffolding the child's learning. Vygotsky's Zone of Proximal Development states that children learn the most when they are able to achieve success with just a bit of a struggle. Achieving success is important, and having that little bit of a struggle is important as well. How a teacher words the problem is able to customize the problem to the child's individual Zone of Proximal Development.

In addition to joining problems, or addition problems that involve action, there are levels of difficulty, based on wording, for subtraction problems that involve action, also called separating problems, and for non-action problems such as those that have two parts joining into a whole, and problems that compare quantities of two parts. With Cognitively Guided Instruction, first graders are able to apply their own strategies to multiplication and division problems, and even problems that involve fractions. (Carpenter, Fennema, Franke, Levi, & Empson, 1999).

Cognitively Guided Instruction is grounded in the philosophy of constructivism. The word that best describes a constructivist mathematics classroom is energy…Teachers in constructivist classrooms *direct* this energy by engaging students actively in the learning process" (Crawford & Witte, 1999). Constructivist classrooms involve *participation* in hands-on activities, *sharing* and discussion with other students, *cooperative* group work in addition to independent work, a high level of *personal interest*, a fostering of *self-confidence*, and a "climate of *community*" (Crawford & Witte, 1999). The philosophy of constructivism is that "teaching and learning occur best in context" (Crawford & Witte, 1999). What that means is that learning is more likely to occur, and be retained, when it is personally meaningful, when it is achieved through "exploration, discovery, and invention" (Crawford & Witte, 1999), when it is involves problem-solving, when it employs the Scientific Method and the students' own data, when it is applied to something useful, when it is shared with others, and when it is transferred to a new context (Crawford & Witte, 1999). Constructivist classrooms are motivating because they "evoke curiosity and emotions" (Crawford & Witte, 1999), and because they include "creativity and joy…laughter, engagement, attention, (and) imagination" (Crawford & Witte, 1999). They are also motivational because they provide a "reason for learning" by demonstrating the personal applications and usefulness to real life (Crawford & Witte, 1999).

What I find so exciting about CGI is that it achieves all of the teaching strategies recommended by Crawford and Witte. The teacher has the ability to come up with problems that relate to the life experiences of the children. In so doing, the child is able to connect a "new concept to something completely familiar" (Crawford & Witte, 1999) and experience the " 'aha' sensation that often accompanies the insight" (Crawford & Witte, 1999). Because sharing individual strategies is an integral part of CGI, children "learn to value the opinions of others" (Crawford & Witte, 1999). Listening to others is a form of gathering data that can help children to reassess their own hypotheses. I love the way that constructivism applies the Scientific Method to real life!

What is the Scientific Method? It starts with being curious about something, having a question. Considering the question in the context of what one already knows, the person forms a hypothesis – a best guess answer. Next the person wants to test his/her hypothesis. To do this, he/she forms a plan for how to test it and determines what materials will be required to do the experiment. After doing the experiment, he/she analyzes the results and draws a conclusion about his/her original hypothesis.

It's the same blueprint for how a teacher designs a lesson plan. The teacher has a hypothesis for how to best teach the lesson, and that hypothesis is stated in his/her objective. The teacher determined what materials he/she will need to do the lesson. The teacher comes up with a plan, or a procedure, for the lesson. In a way, the lesson is the experiment. Finally, the teacher assesses the results and determines if her objectives were met, and if she should alter her hypothesis and try the experiment differently in the future.

To me, CGI is to math what Inquiry Learning, another application of constructivism, is to education in general. Both processes follow the Scientific Method. The process of Inquiry Learning is to identify a problem or question of personal interest and form a hypothesis about it, to develop a plan for how to explore it, to interact with others while collecting data and information, to assess what one discovers from one's own research as well as from what others have discovered, and to draw a conclusion about one's original hypothesis. The goal of Inquiry Learning is "to identify questions;

explore possibilities; construction explanations, interpretations, or solutions; and then evaluate and revise their initial thinking…It puts students in the driver's seat of learning as they consider alternative and sometimes competing explanations, solutions, or hypotheses; debate investigative methods; argue about interpretations; and defend conclusions or proposed solutions" (Fleming, 1999). As opposed to learning *about* things, Inquiry Learning allows children to learn *as* a scientist, *as* a detective, *as* an author, *as* an inventor, *as* an explorer, *as* a historian. In real life, questions are often open-ended. There are often many possible answers, with many possible strategies for getting the answers. Inquiry Learning is about getting people to think, to focus on the process of coming up with an answer, rather than on the answer itself. The philosophy behind Inquiry Learning is that reflection opens the door to understanding, and that understanding allows for transfer of what one learns now to new situations later.

Like Inquiry Learning, CGI focuses not on "finishing the assigned task (as much as) making sense of, and communicating about, mathematics" (Clements, 1997). The focus is on where so much of the learning takes place – the process. Also like Inquiry Learning, CGI definitely puts the student in the driver's seat. The student finds the problem meaningful. The student is actively engaged in trying out his/her hypothesis of how to solve the problem, of listening to other students explain their strategies and assessing that new information, and of drawing conclusions. Throughout the whole process, the student is making connections between what one has already known, what one experiences during the process, what one hears from others, and what one tries applying to other situations. In a way, all of learning follows this process. It makes complete sense, then, that how one learns mathematical concepts, patterns, and skills will follow this process. It is very exciting!

Because CGI uses constructivist strategies and acknowledges the real life application of the Scientific Method, it meets all the process standards as set by the National Council of Teachers of Mathematics. It requires students to “hypothesize, predict, observe, and reason about mathematical situations” (Goldsmith & Mark, 1999). In so doing, it covers problem solving, reasoning, communication of that reasoning, connections to be made between what one knows and what one hears from others, and a flexibility of strategies to be represented. As emphasized by the standards, it focuses on conceptual understanding rather than on rote memorization. It values the memorization of basic facts, but in the context of meaningful exercises. It sees the incorporation of basic facts into one’s problem solving strategies as a natural progression in cognitive development. Rather than learning basic facts solely from drills, it provides the opportunity for basic facts, as well as strategic thinking and number sense, to develop from the playing of games (Goldsmith & Mark, 1999). Standards-based curriculum also requires “multiple entry points” for children of different cognitive developmental levels and learning styles (Goldsmith & Mark, 1999). The ability to use teacher’s language as a scaffolding tool makes CGI adaptable to any cognitive developmental level. The same problem can be solved on so many different levels, from direct modeling to algebraically, that it is able to challenge a large range of ability levels. Finally, having the children explain their strategies provides a “window into the children’s thinking” (Jenkins, 1998). It makes possible a continuous assessment of the child’s cognitive understanding and strategic skills, and of how the teacher can adjust his/her “subsequent instructional decisions” (Jenkins, 1998) to better meet the needs of the children.

Learning about Cognitively Guided Instruction has been personally meaningful and exciting for me. For every story problem posed, I found myself trying to solve it by the variety of strategies exemplified. I lost count of how many times I said “aha!” It made me think about how I solve problems, and the variety of strategies I use. I love listening to the strategies that others come up with for solving problems. In my view, Cognitively Guided Instruction is grounded in the philosophy of constructivism, based on the cognitive developmental stages of Piaget, and supportive of NCTM’s process standards for mathematics. It is fun, personally meaningful, and engaging. It treats the children as problem solvers – which is exactly how they think of themselves. Not only do I like CGI for students, but I like it for myself as a teacher. It helps me to assess the children’s ability levels, and offers me that window into the children’s thinking. As a primary grade teacher, I am fascinated by how children think!

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